CS 126 Lecture T3: Formal Languages

Outline

- **Introduction**
- Defining grammar
- Type 3 grammars
- Type 2 grammars
- Chomsky hierarchy
- Conclusions

Where We Are

- T1
  - Simplest language generators: regular expressions
  - Simplest language recognizer: FSAs
- T2: more powerful machines
  - FSA, NFSA
  - PDA, NPDA
  - TM
- **T3: more powerful languages associated with the more powerful machines**
- T4:
  - Nature of most powerful machines
  - Languages that no machine can ever deal with

Review: Formal Languages

- Formal definitions
  - An **alphabet**: a finite set of symbols
  - A **string**: a finite sequence of symbols from the alphabet
  - A **language**: a (potentially infinite) set of strings over an alphabet
- Intriguing topic: **finite representation** of a language
  - How?
    + language **generators** (a set of rules for producing strings)
    + language **recognizers**

Languages and Automata

Given a RE, can construct a FSA that recognizes any string matching the RE

Given a FSA, can construct a RE that matches the strings recognized by the FSA

Thus, FSA's and RE's computationally equivalent

More powerful machines than FSAs:

- PDA: add pushdown stack, nondeterminism
- LBA: use (linear-bounded) tape
- Turing machine: use infinite tape

What languages do these machines recognize?

Why Learn Languages?

Concrete applications:

- understanding computability
- compiler implementation
- language recognition/translation
- models of physical world

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Grammars

- Generate words in a formal language by a process of replacing symbols systematically

Four elements:
- nonterminals
- terminals
- start symbol
- productions

NONTERMINAL symbols
- "local variables" for internal use
- notation: <name>

TERMINAL symbols
- set of characters that appear in the words
- "alphabet" of the language
- ex: {0,1} or ASCII

START symbol
- one particular nonterminal

PRODUCTIONS
- replacement rules
- ordered pairs of strings of symbols
- notation (ex.): a<B>c -> <D>e<F>

Example 1

Nonterminal:

<sentence> <subject> <verb> <object>

Terminal:

horse dog cat saw heard the

Start:

<sentence>

Example 1 (cont.)

Productions:

```
<sentence> -> <subject><verb><object>
<subject>  -> the horse
<subject>  -> the dog
<subject>  -> the cat
<object>   -> the horse
<object>   -> the dog
<object>   -> the cat
<verb>     -> saw
<verb>     -> heard
```

"Words" in the language:

```
the horse saw the dog
the dog heard the cat
the cat saw the horse
```

Example 2

Nonterminal:

<stmnt> <selection-stmnt> <expression>, etc.

Terminal:

if while else switch ( ), etc.

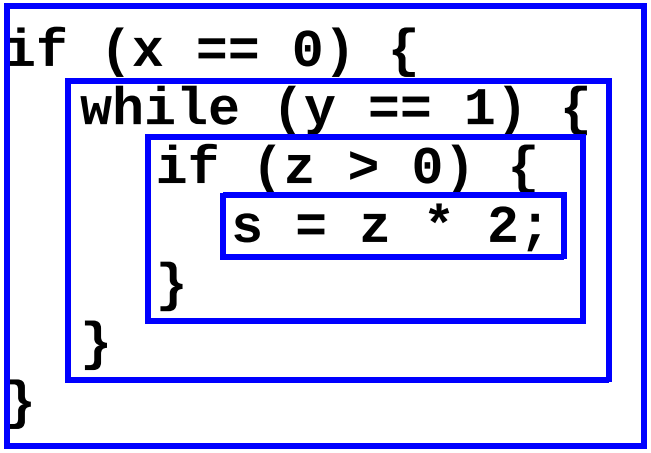
Start:

<translation-unit>

Example 2 (cont.)

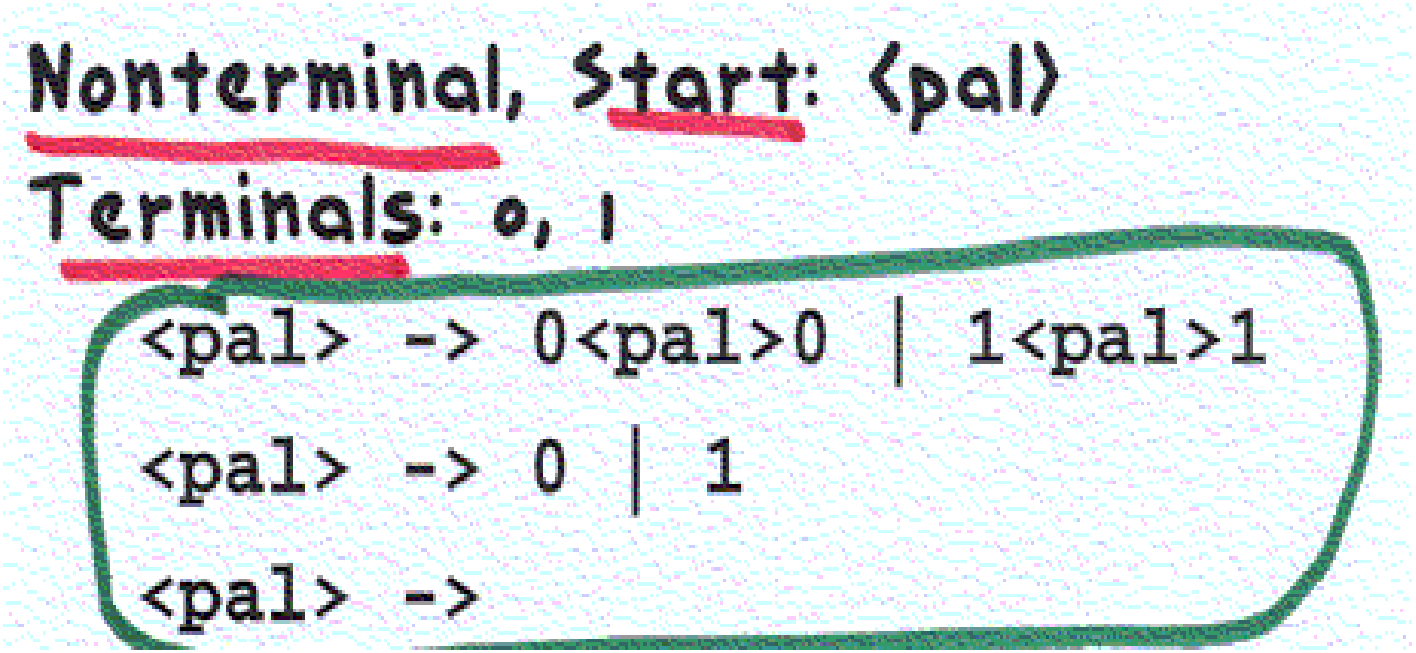
Productions:

```
<stmnt>
    -> <selection-stmnt>| <iteration-stmt> | ...
<selection-stmnt>
    -> if ( <expression> ) <stmnt>
    -> if ( <expression> ) <stmnt> else <stmnt>
<iteration-stmnt>
    -> while ( <expression> ) <stmnt>
    -> do <stmnt> while ( <expression> ) ;
```



Each box is one production.

Example 3

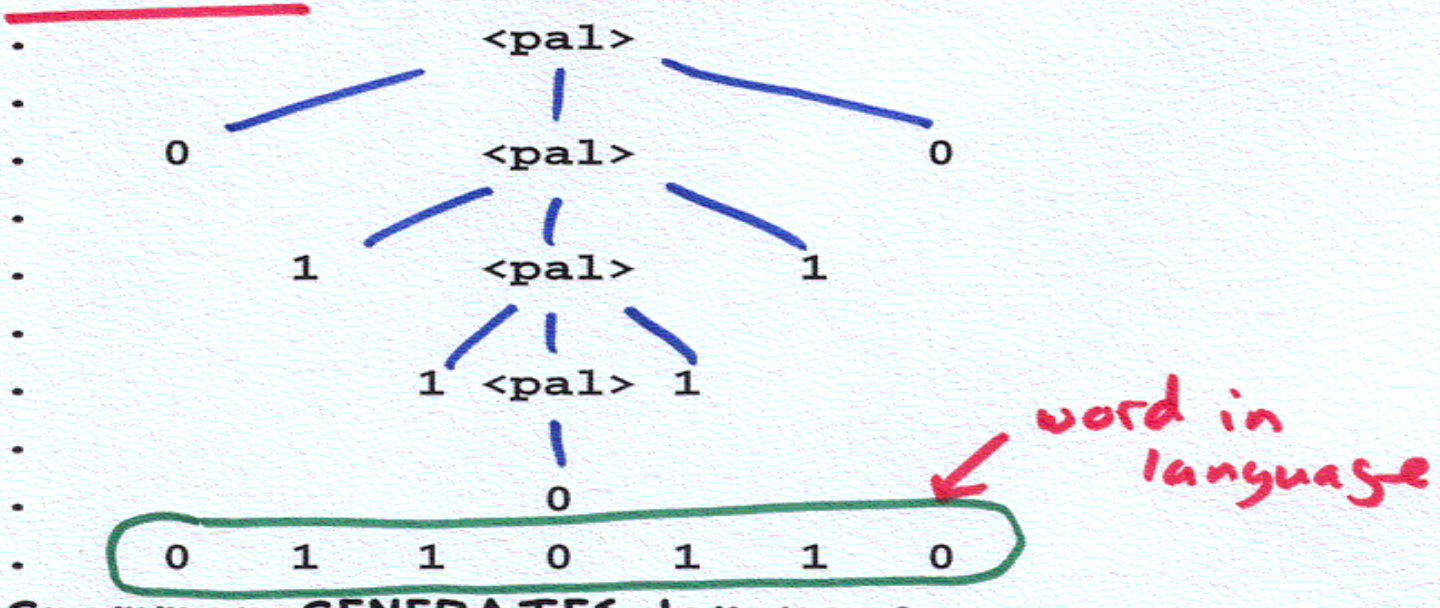

Nonterminal, Start: <pal>
Terminals: 0, 1
<pal> -> 0<pal>0 | 1<pal>1
<pal> -> 0 | 1
<pal> ->

Example 3 (cont.)

DERIVATION: apply replacement rules until no nonterminals left

<pal> -> 0<pal>0 -> 01<pal>10
-> 011<pal>110 -> 0110110

PARSE TREE exhibits derivation



Grammar GENERATES language

set of all strings that could be derived

0, 1, 00, 11, 010, 101, 000, 111, 0000, 00100

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Type 3 Grammars

Restrict all productions to be of the form

<X> -> x

<X> -> <Y>x

Ex:

<A> -> <z>0

<z> -> <A>1

<z> ->

generates alternating sequence of 0's and 1's

Type 3 Grammars and Regular Expressions

THM: Type 3 grammars are equivalent to REs

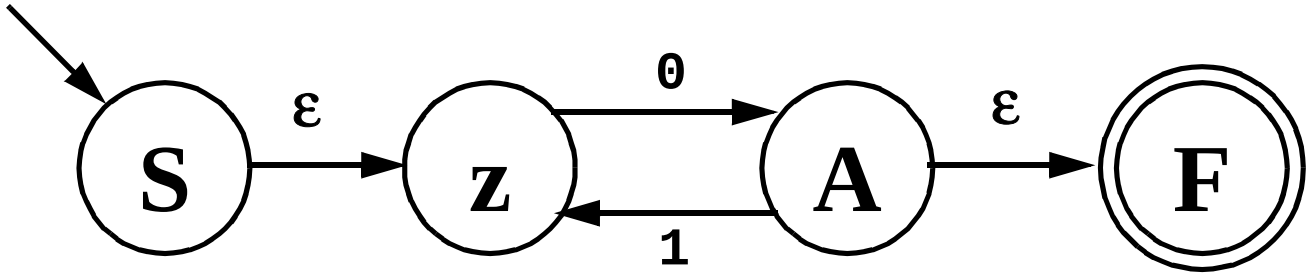
proof sketch:

- given a Type 3 grammar, construct an FSA that recognizes any string in the language generated by the grammar
- given an FSA, construct a Type 3 grammar that generates the strings recognized by the FSA

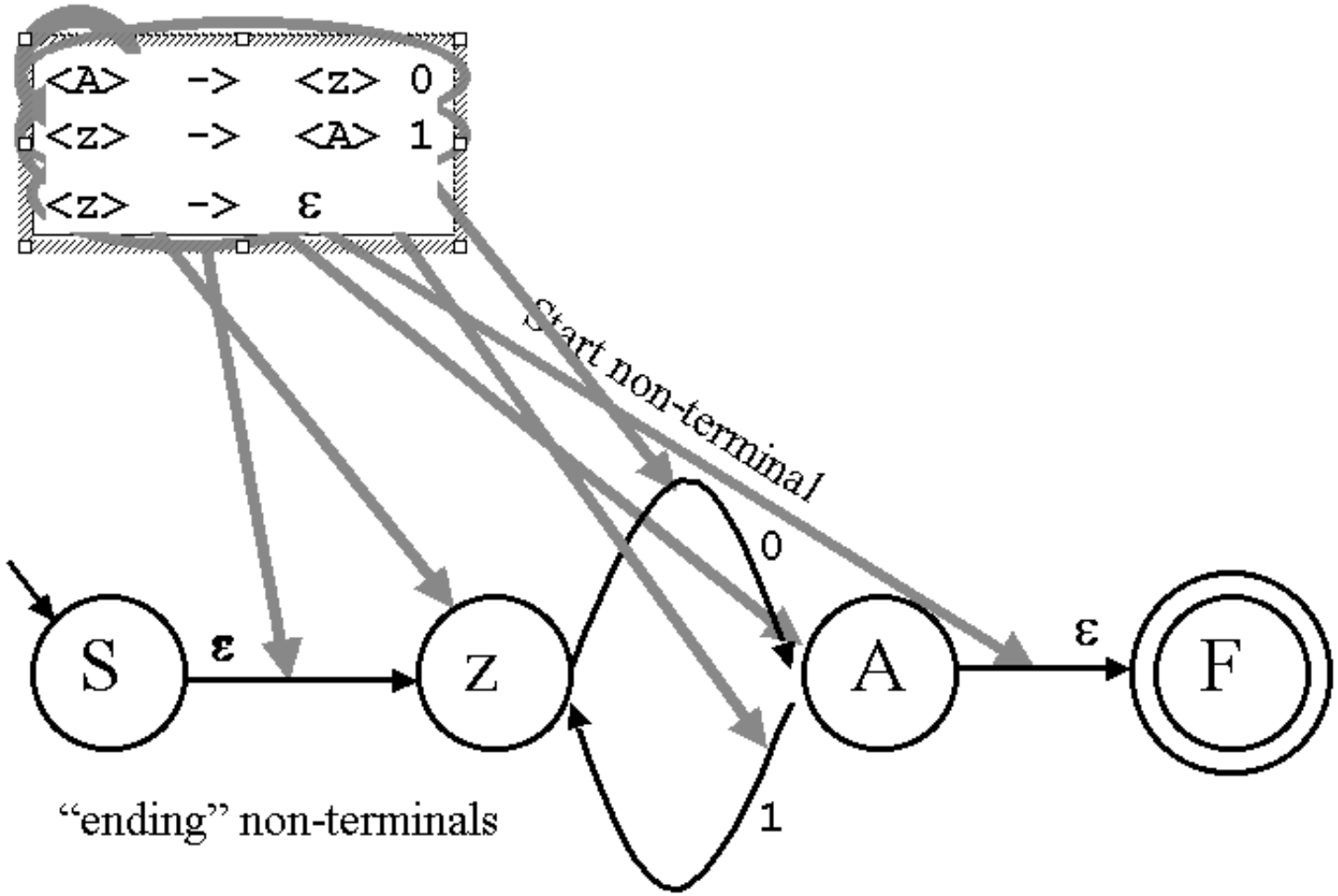
[FSA states correspond to nonterminals]

Example

```
<A> -> <z>0
<z> -> <A>1
<z> -> ε
```



Demo: Construction of Equivalent FSA



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Context-Free Grammars

Restrict all productions to allow only a single nonterminal on the LHS

Ex:

```
<pal> -> 0<pal>0
<pal> -> 1<pal>1
<pal> -> 0
<pal> -> 1
<pal> ->
```

Ex:

grammar for C

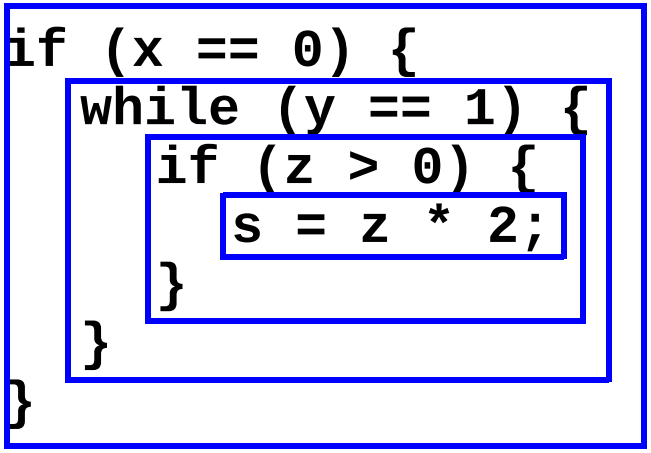
Also called CONTEXT-FREE grammars

Much more descriptive than regular expressions

Example Context-Free Grammar

Productions:

```
<stmnt>
    -> <selection-stmnt>| <iteration-stmt> | ...
<selection-stmnt>
    -> if ( <expression> ) <stmnt>
    -> if ( <expression> ) <stmnt> else <stmnt>
<iteration-stmnt>
    -> while ( <expression> ) <stmnt>
    -> do <stmnt> while ( <expression> ) ;
```



Each box is one production.

How to Recognize Context-Free Grammars

Q. How does the FSA have to be augmented to recognize strings from languages generated by context-free grammars?

C compiler

A. Add a "memory" capability (as expected) *and* add power of nondeterminism (!)

Still limited: can't generate strings with equal numbers of a's, b's, and c's

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Type 1 Grammars (Context-Sensitive) and Type 0 Grammars

Type 1 (context-sensitive) grammars:

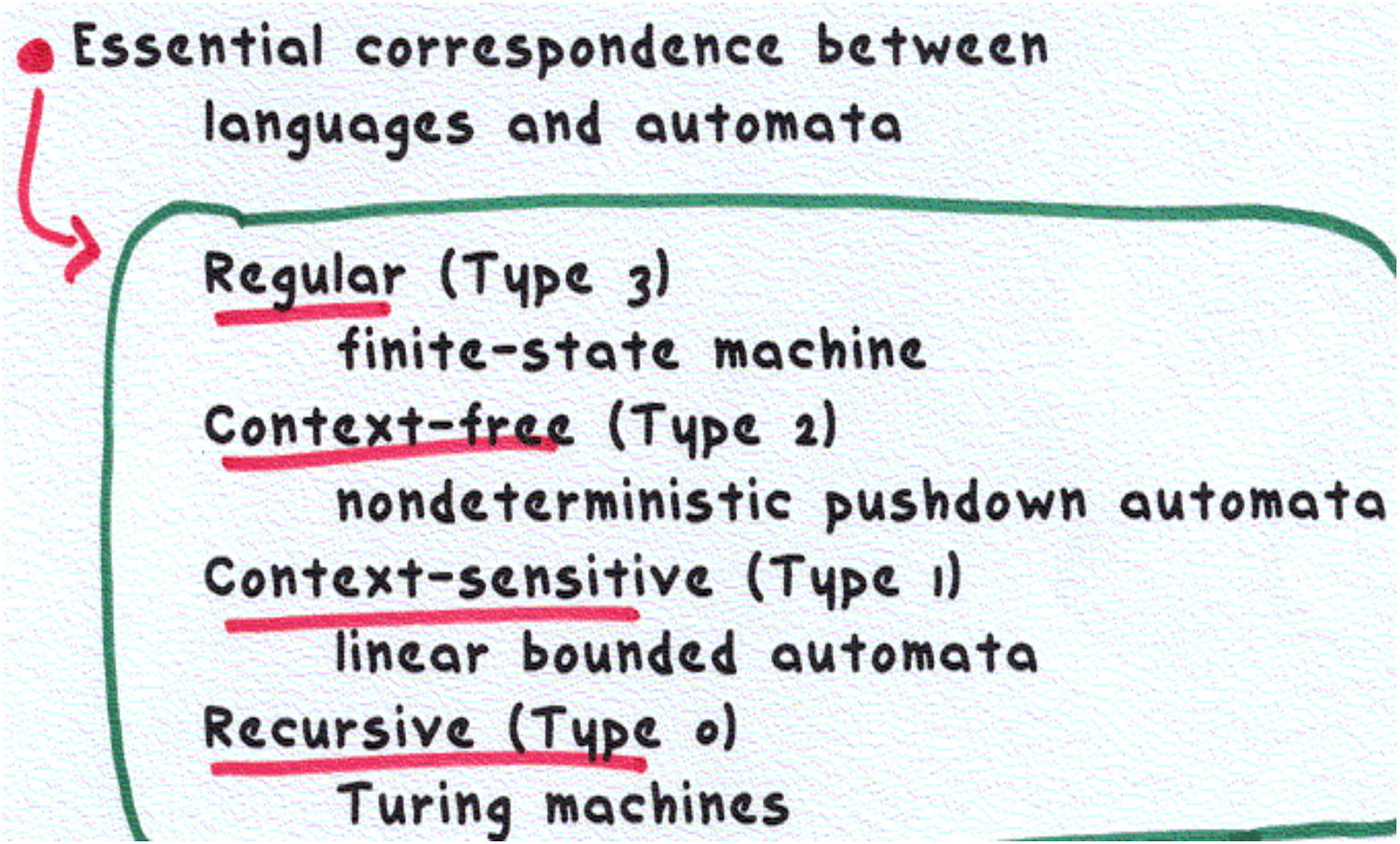
add productions of the type

[X]⟨Y⟩[Z] -> [X]y[Z]

where [X] and [Z] represent any sequence of terminals and nonterminals

Type 0 grammars: no restrictions

Chomsky Hierarchy, Languages and Automata



Language Expressiveness and Machine Power

- One-to-one correspondence to machines
  persists through the hierarchy
- Each type is more descriptive than the previous
- Each machine is more powerful than the previous

Are there limits to machine "power"?
Are there languages
that no machine can recognize?
[stay tuned]

$T_{3.8}$

Example Language Outside Chomsky Hierarchy: Lindenmayer Systems

Apply productions simultaneously

Ex:

```
0 -> 1 [ 0 ] 1 [ 0 ] 0
1 -> 1 1
```

Reproduction

Growth

Start with o

At stage i, apply rules to each symbol in string form stage i-1

```
0
1 [   0   ]1 [   0   ]   0
11[1[0]1[0]0]11[1[0]1[0]0]1[0]1[0]0
```

Graftal Plants: Lindenmayer in 2D

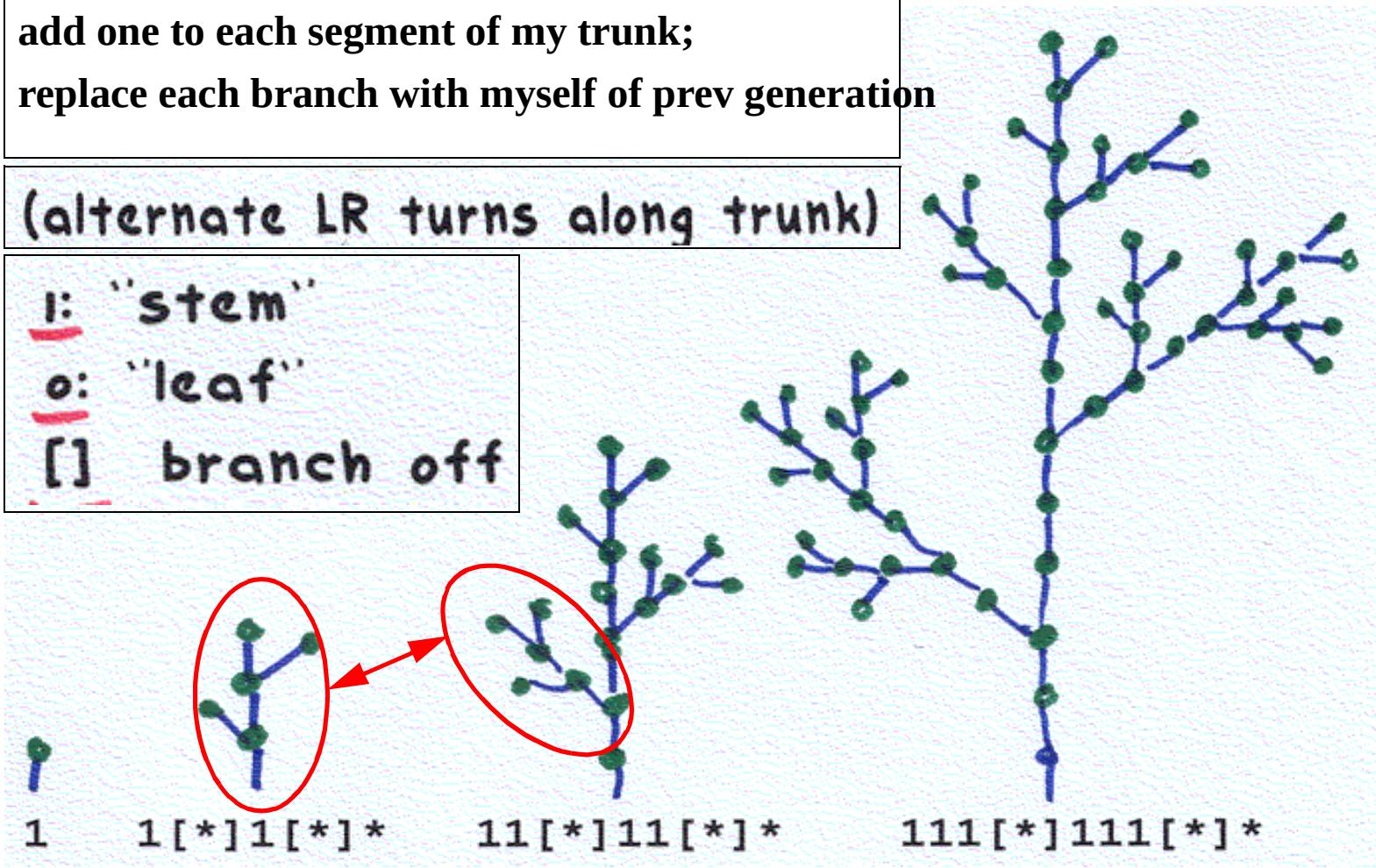
"Production" rules:

add one to each segment of my trunk;

replace each branch with myself of prev generation

(alternate LR turns along trunk)

1: "stem"
0: "leaf"
[] branch off



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More on Languages, Machines, and Life

- The fine art of describing infinite beauty and variety with finite representation
  - Shakespearean sonnets and machines that can identify authorship?
    (Is intelligence/art nothing but a set of rules, with some non-determinism thrown in?)
  - DNA: the language of life
    (Is life nothing but a set of rules, with some non-determinism thrown in?)
  - .....

What We Have Learned Today

- What is a grammar?
- What is a type 3 grammar? type 2? type 1? type 0?
- How to construct an FSA from a type 3 grammar?
- What is the Chomsky Hierarchy?
- What type of automata recognize which grammars?