CS 126 Lecture P7:
Trees

First Midterm

- When: 7pm, 10/20 (Wednesday)
- Where: MC46 (here)
- What: lectures up to (and including) today's
- Format: close book, minimum coding
- Preparation: do the readings and exercises

Why Learn Trees?

Culmination of the programming portion of this class!

- Comparison against **arrays** and **linked lists**
- Trees -- a versatile and useful **data structure**
- A naturally **recursive** data structure
- Applications of **stacks** and **queues**
- Reinforce our **pointer manipulation** knowledge

Outline

- **Searching and insertion *without* trees**
- Searching and insertion *with* trees
- Traversing trees
- Conclusion

- Class list

192-034-2006	Alam
201-212-1991	Baer
202-123-0087	Bagyenda
177-999-9898	Balestri
232-876-1212	Benjamin
122-999-3434	Berube
...	

- Desired operations
  - add student
  - return name, given ID number

SEARCH KEY

- Similar applications
  - online phone book
  - airline reservations
  - "symbol table"
  - ...

GOAL: fast search *and* insert
even for huge databases

Encapsulating the Item Type Stored

- Define "Item.h" file to encapsulate item type

```
typedef int Key;
typedef struct{ Key key; char name[30]; } Item;
Item NULLitem = { -1, ""}
```

- A single item itself is an ADT
- So we don't see the internals of the item type when we implement searching and insertion
- So our code will work for <u>any</u> item type

Array Representation: Binary Search

```
Item items[13];
```

Ex: search for 25

Index	0	1	2	3	4	5	6	7	8	9	10	11	12	
Keys	06	13	14	25	33	43	51	53	64	72	84	97	99	← 1st step
.	06	13	14	25	33	43	← 2nd step							
.				25	33	43	← 3rd step							
.				25			← 4th step							

- Keep array of Items, in sorted order
- Use bisection method to find Item sought

[See also Lecture P6; Programs 2.2 and 12.6]

Array Representation: Binary Search

```
Item search(int l, int r, Key v)
  { int m = (l+r)/2;
    if (l > r) return NULLitem;
    if (v == st[m].key) return st[m];
    if (l == r) return NULLitem;
    if (v < st[m].key)
      return search(l, m-1, v);
    else return search(m+1, r, v);
  }
```

Cost of Binary Search

Q: How many "comparisons" to find a name?

A: lg N

divide list in half each time

Ex: 5000 -> 2500 -> 1250 -> 625 -> 312 -> 156 -> 78 -> 39 -> 18 -> 9 -> 4 -> 2 -> 1

log N = number of digits in decimal rep. of N

lg N = number of digits in binary rep. of N

$\log_2 N \equiv \lg N$

lg(thousand) = 10

lg(million) = 20

lg(billion) = 30

$N=2^x, \quad x=\log_2 N$

Without binary search, might have to look at everything, so savings is substantial for very large files.

Insertion into Sorted Array

Problem: insert operation is usually slow

Ex: to insert 49

.	0	1	2	3	4	5	6	7	8	9	10	11	12
.	06	13	14	25	33	43	51	53	64	72	84	97	99

have to move larger keys over one position

.	0	1	2	3	4	5	6	7	8	9	10	11	12	13
.	06	13	14	25	33	43	49	51	53	64	72	84	97	99

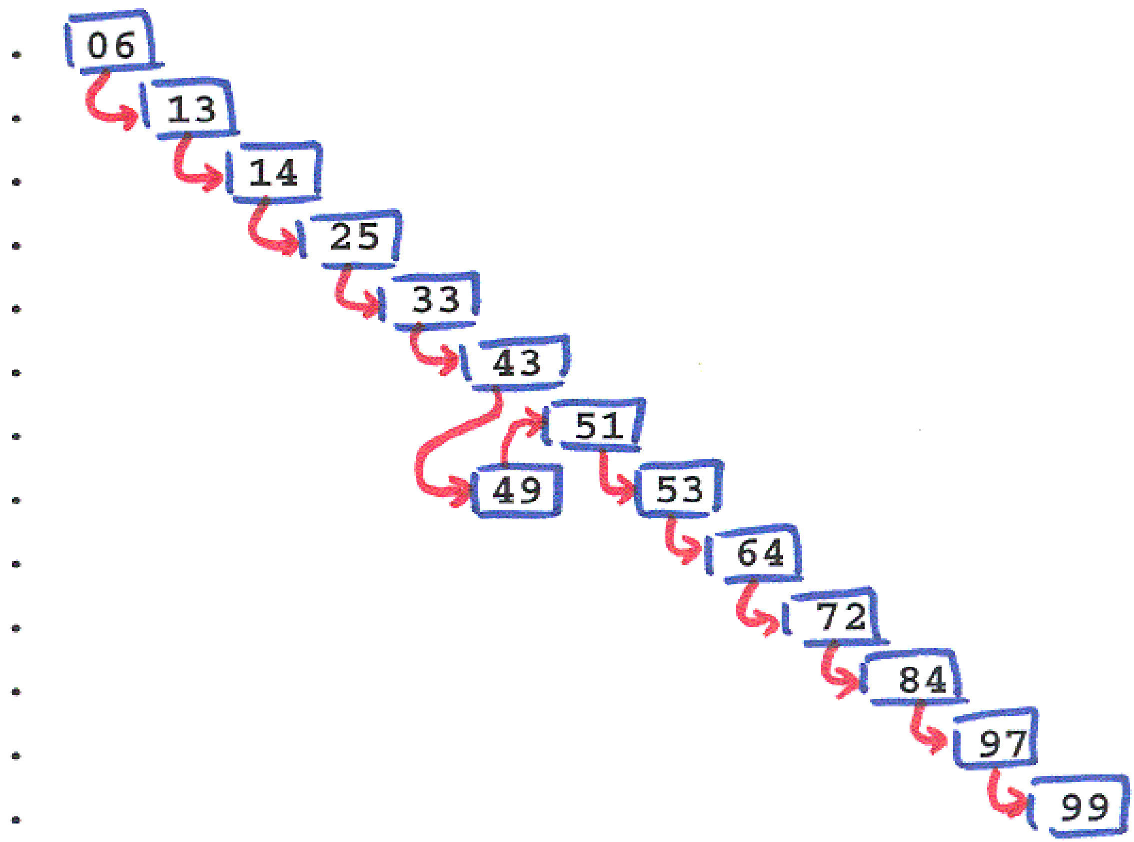
Linked List Representation

Keep items in a linked list

```
typedef struct STnode* link;
struct STnode { Item item; link next; };
```

Inserting into Linked List

- Advantage of linked representation
  can insert just by changing links
  (no need to "move" anything)



Exercises and Summary

- Assuming a sorted linked list, try writing code for
  - both searching and insertion
  - using both loop and recursion
- Summary so far:

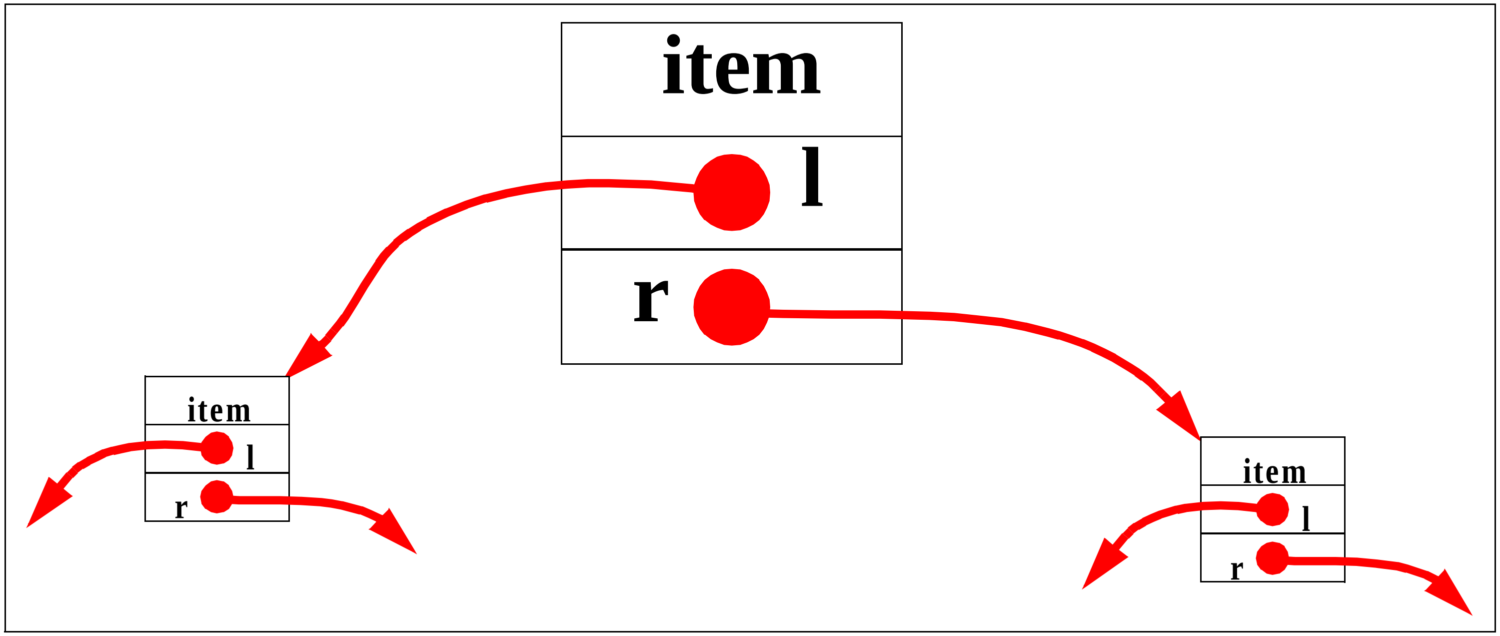
ARRAY: fast search, slow insert

LINKED LIST: slow search, fast insert

Outline

- ~~Searching and insertion *without* trees~~
- **Searching and insertion *with* trees**
- Traversing trees
- Conclusion

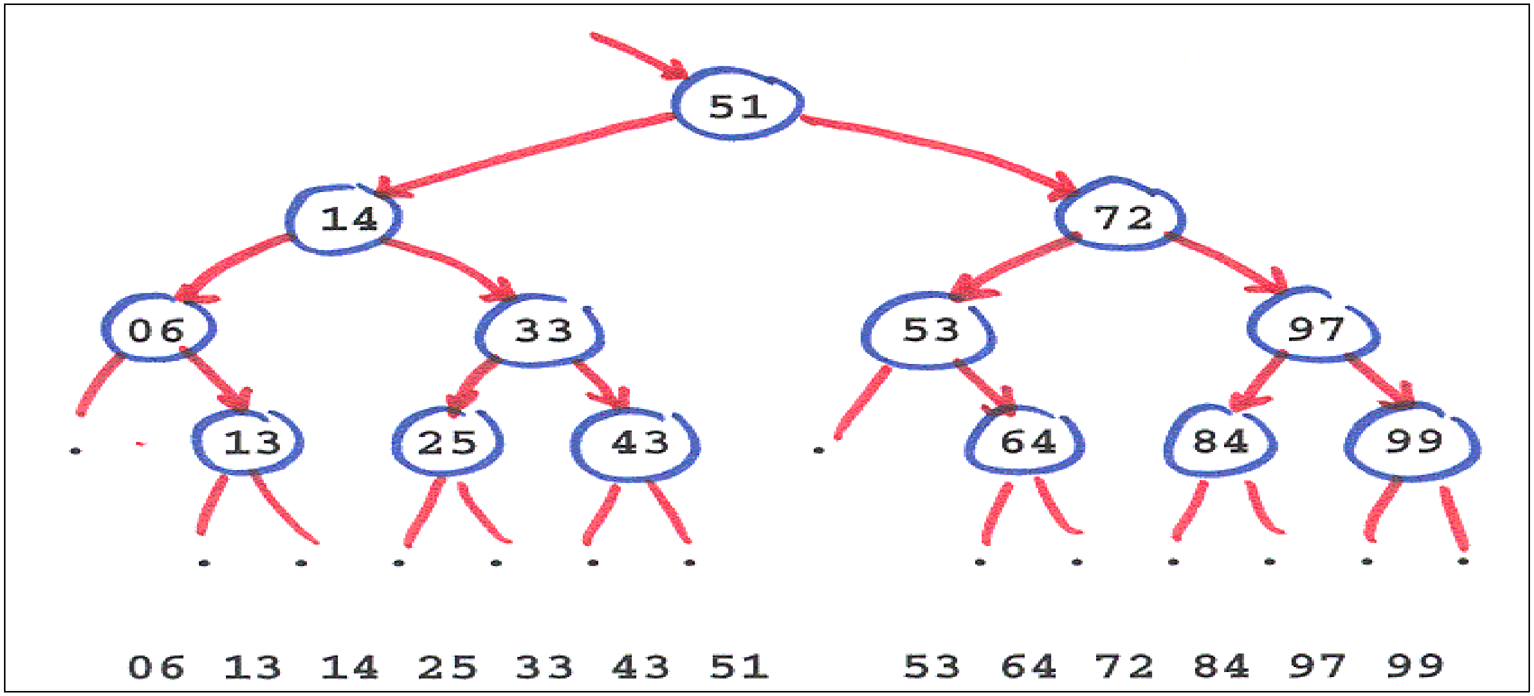
Declaring a Tree Type



- Use *two* links per node

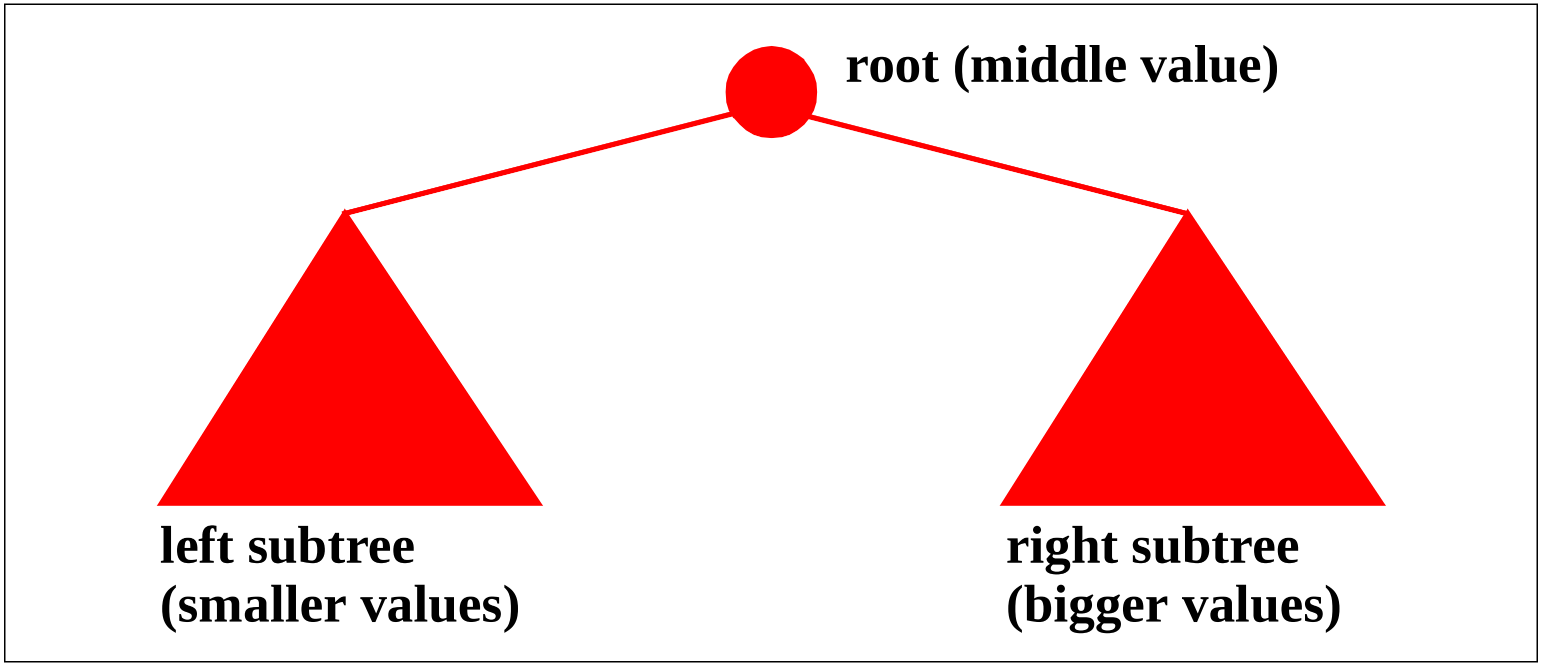
```
typedef struct STnode* link;
struct STnode { Item item; link l, r; };
```

Binary Tree



- Think of keys printed in order, left to right
  - take middle name for top, or "root" node
  - build tree recursively
    - "l" points to tree for left half
    - "r" points to tree for right half
- NULL links at bottom: "no information here"

Binary Search Tree Property



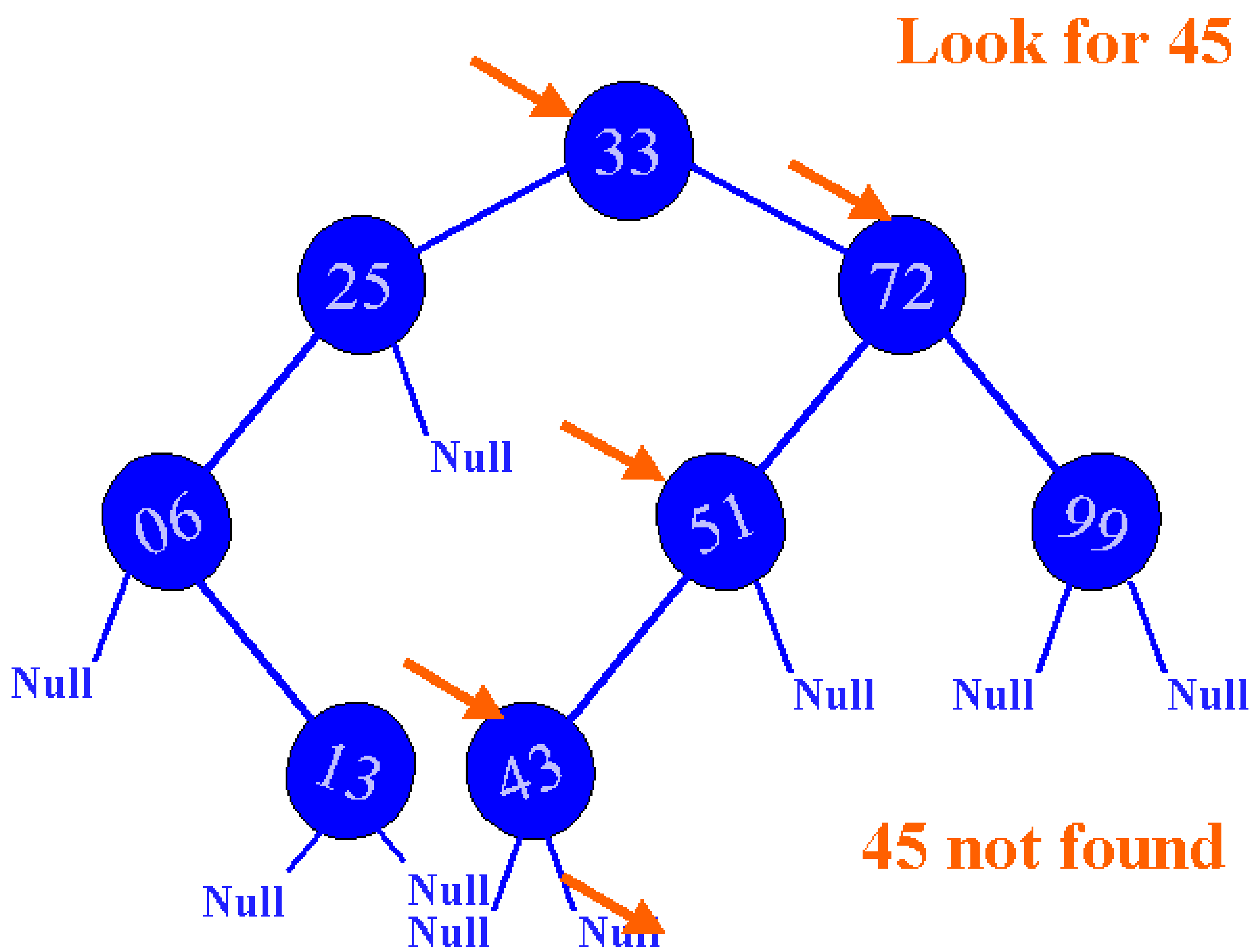
- Maintain ordering property for **all** subtrees
- Must maintain ordering property at all times (just like we keep an array or linked list sorted at all times)

Searching in Binary Search Tree

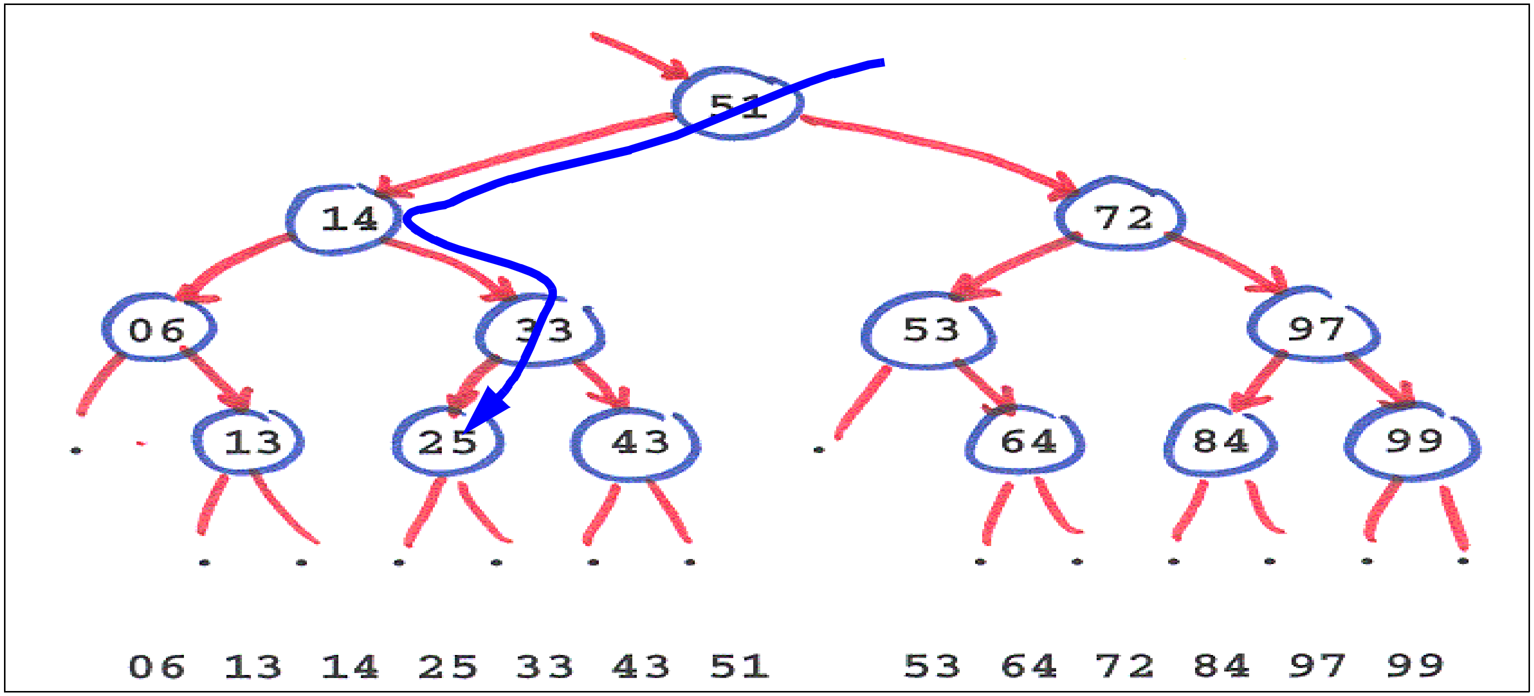
```
Item searchR(link h, Key v)
  {
    if (h == NULL) return NULLitem;
    if (v == h->item.key) return h->item;
    if (v < h->item.key)
      return searchR(h->l, v);
    else return searchR(h->r, v);
  }
Item STsearch(Key v)
  { return searchR(head, v); }
```

- Start at "head", link to the root
  - if current node has key sought, return
  - go left if key < key in current node
  - go right if key > key in current node

Search Demo


Look for 45
33
25
72
Null
06
51
99
Null
Null
Null
Null
13
43
45 not found
Null
Null
Null
Null

Search Cost



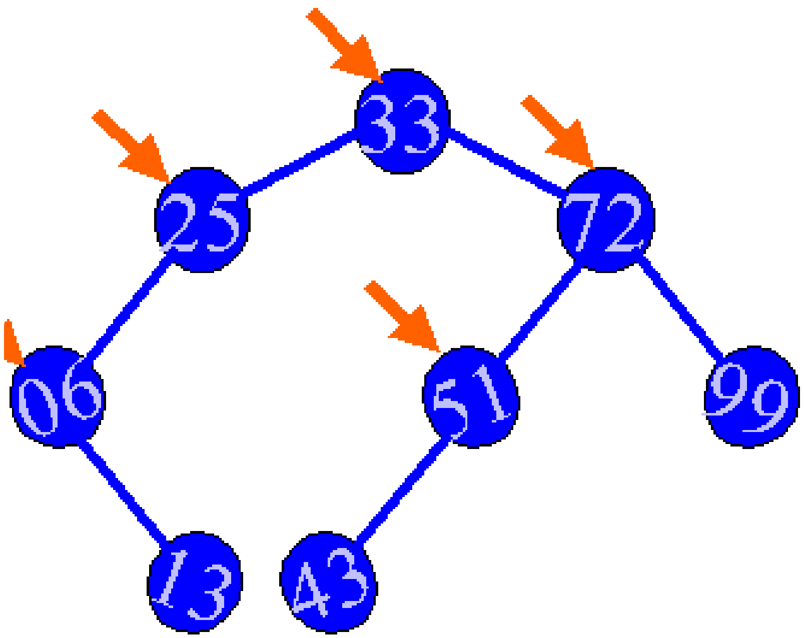
- Nodes examined on the search path roughly correspond to nodes examined during binary searching an array
- So the cost is same as binary searching an array (lg N)
- That is **<u>if</u>** the tree is balanced

Insertion into Binary Search Trees

```
link NEW(int item, link l, link r)
  { link x = malloc(sizeof *x);
    x->item = item; x->l = l; x->r = r;
    return x;
  }
link insertR(link h, Item item)
  { Key v = key(item);
    if (h == NULL)
      return NEW(item, NULL, NULL);
    if less(v, key(h->item))
      h->l = insertR(h->l, item);
    else h->r = insertR(h->r, item);
    return h;
  }
void STinsert(Item item)
  { head = insertR(head, item); }
```

- Search for key not in tree
  - ends on a NULL pointer
  - node "belongs" there
  - make a node, link it into the tree

Insertion Demo




```
Link
insert(Link h, Item it) {
  if (h == NULL)
    return newLeaf(it);
  if (less(key(it),
              key(h->item)))
    h->l = insert(h->l, it)
  else
    h->r = insert(h->r, it)
  return h;
}
```

More Notes on Binary Search Tree Insertion

```
link insertR(link h, Item item)
  { Key v = key(item);
    if (h == NULL)
      return NEW(item, NULL, NULL);
    if less(v, key(h->item))
      h->l = insertR(h->l, item);
    else h->r = insertR(h->r, item);
    return h;
  }
```

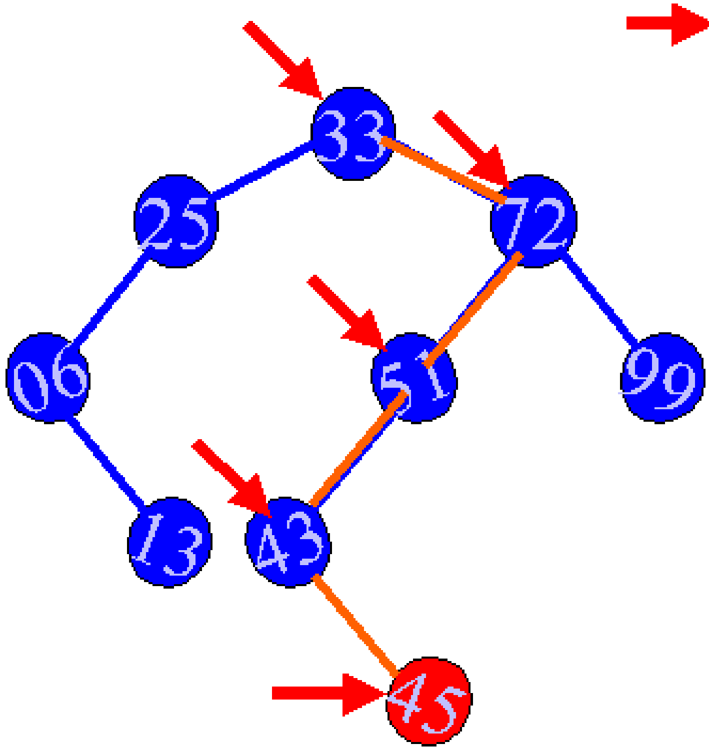
attach the "new" subtree to the current root

subtree containing the new value

- Each recursive call returns the root pointing to the subtree with the new value already inserted
- Do this for base case and inductive case

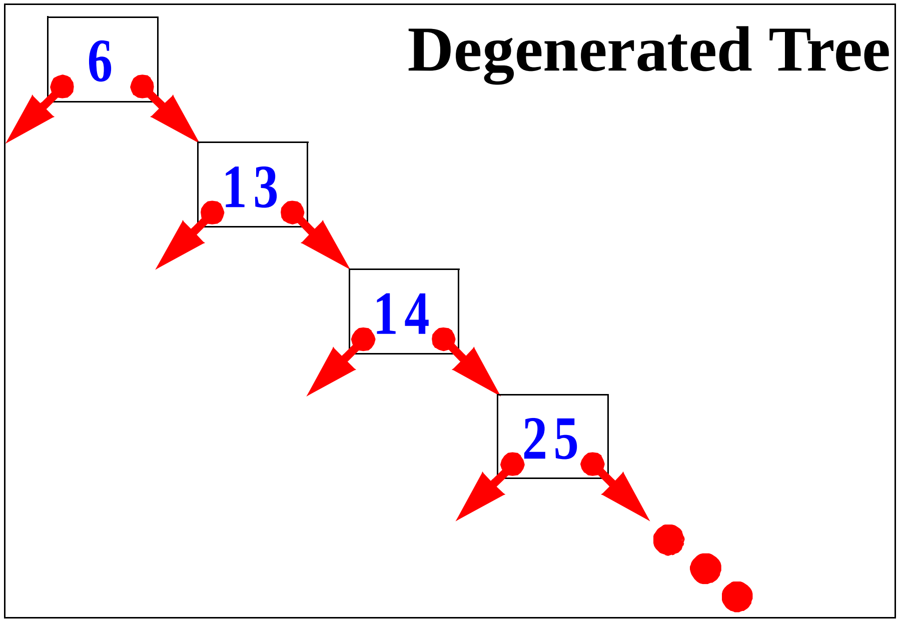
Another Insertion Demo

Insert 45




```
Link
insert(Link h, Item it) {
  if (h == NULL)
    return newLeaf(it);
  if (less(key(it),
             key(h->item)))
    h->l = insert(h->l, it);
  else
    h->r = insert(h->r, it);
  return h;
}
```

Insertion Cost



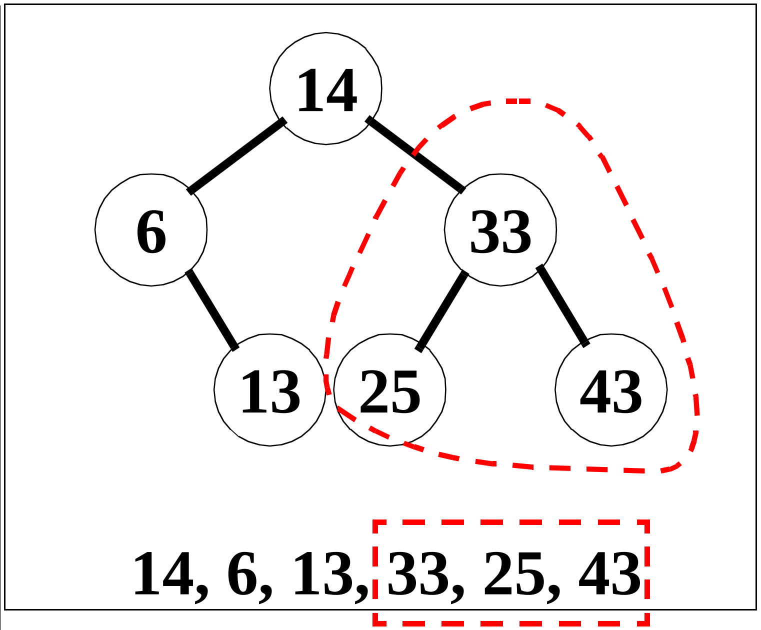
- "Normally", insertion is like search, so similar cost. But...
- Tree shape depends on key insertion order
  - sorted, reverse: degenerates to linked list
  - "random": avg. dist. to root is about 1.44 lg N

Outline

- ~~Searching and insertion *without* trees~~
- ~~Searching and insertion *with* trees~~
- **Traversing trees**
  - Goal: “visit” (process) each node in the tree
- Conclusion

Preorder Traversal

```
visit(link h) {
  printf("%d %s ",
         h->item.ID,
          h->item.name);
}
traverse(link h) {
  if (h != NULL) {
    visit(h);
    traverse(h->l);
    traverse(h->r);
  }
}
```



- Visit before recursive calls
- Generalizes to any tree: depth-first-traversal

Traversing Binary Trees

Goal: "visit" (process) each node in the tree

```
visit(link h)
  { printf("%d %s ", h->item.ID, h->item.name);
traverse(link h)
  {
    if (h != NULL)
      {
        traverse(h->l);     <- preorder
        visit(h);           <- inorder
        traverse(h->r);     <- postorder
      }
  }
```

- Goal realized no matter what order the statements in the "if" are executed

Preorder: visit before recursive calls
Inorder: visit between recursive calls
Postorder: visit after recursive calls

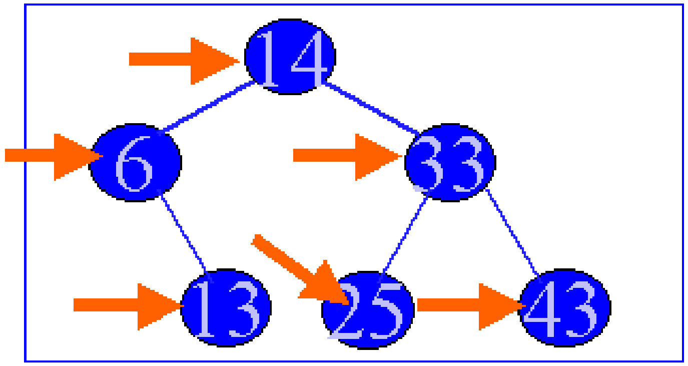
IMPORTANT NOTE:
inorder search provides "free" SORT in binary search trees!

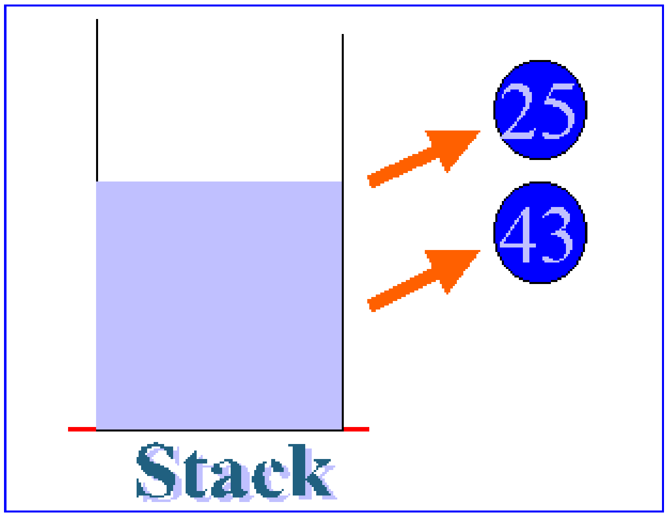
Preorder Traversal with a Stack

- Visit the top node on the stack
  - push its children

```
traverse(link h)
  {
    STACKpush(h);
    while (!STACKempty())
      {
        h = STACKpop(); visit(h);
        if (h->r != NULL) STACKpush(h->r);
        if (h->l != NULL) STACKpush(h->l);
      }
  }
```

Preorder Traversal Demo





14 ,6,13,33,25,43

Output

```
Traverse(Link h) {
  stackPush(h);
  while (!stackEmpty()) {
    h = stackPop();
    printValue(h);
    if (h->r != NULL)
     stackPush(h->r);
    if (h->l != NULL)
     stackPush(h->l);
  }
}
```

Level Order Traversal

- Use a queue instead of a stack

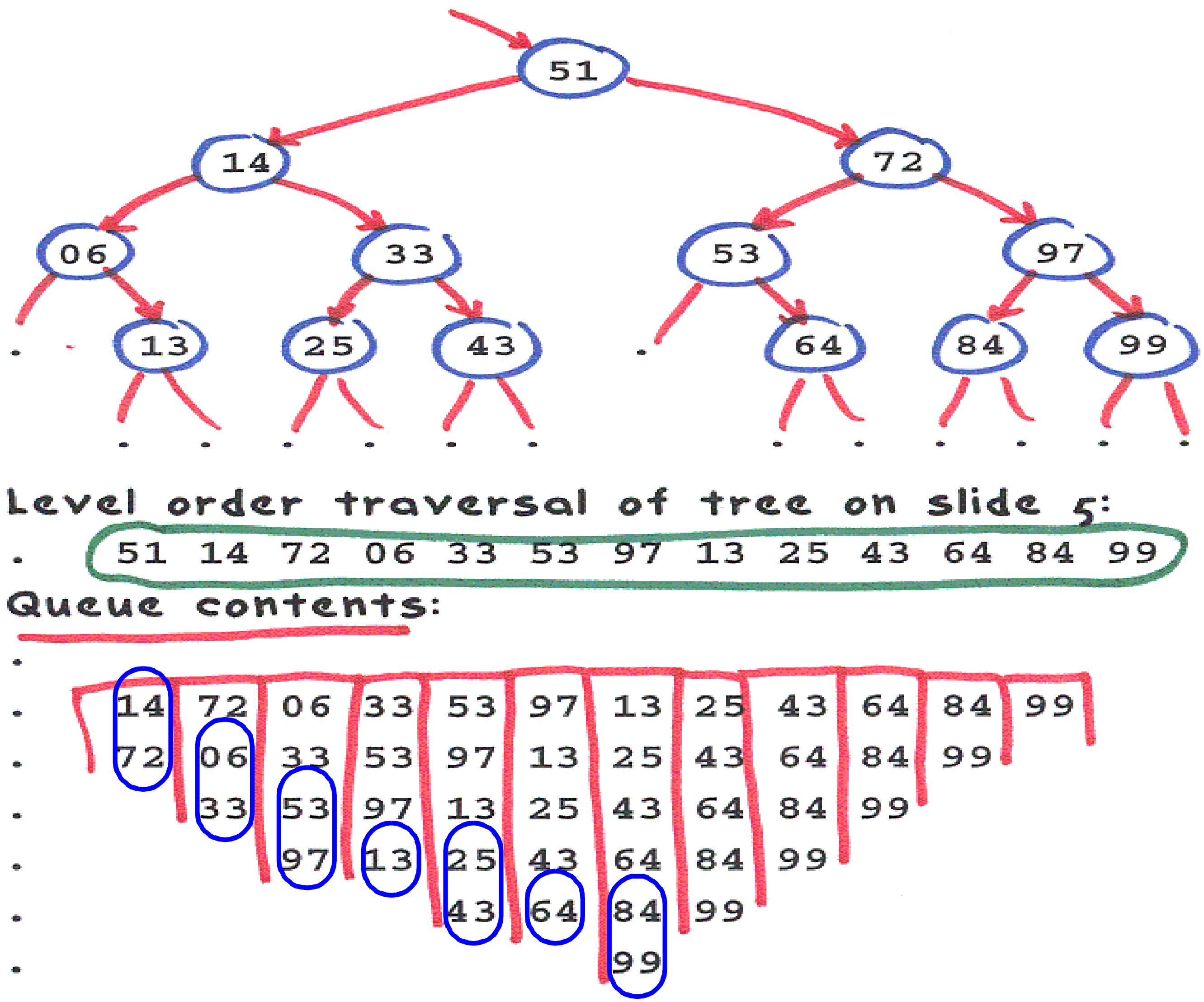
```
traverse(link h)
   {
      QUEUEput(h);
      while (!QUEUEempty())
         {
            h = QUEUEget(); visit(t);
            if (h->l != NULL) QUEUEput(h->l);
            if (h->r != NULL) QUEUEput(h->r);
         }
   }
```

Visits nodes in order of distance from root

- Works for general trees
- Generalizes to BREADTH-FIRST SEARCH in graphs

P7.12

Level Order Traversal Example



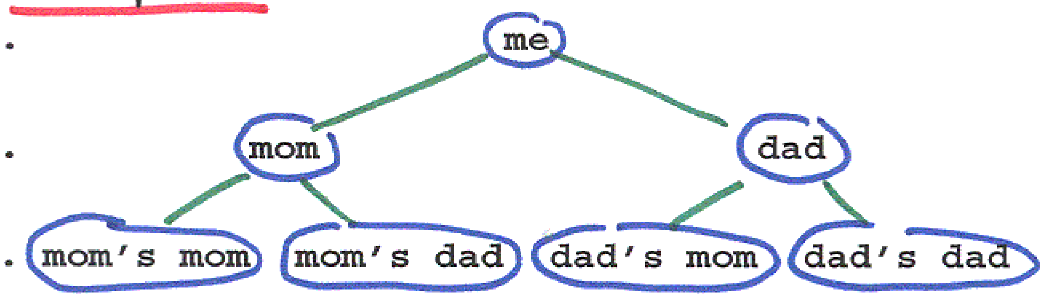
Outline

- ~~Searching and insertion *without* trees~~
- ~~Searching and insertion *with* trees~~
- ~~Traversing trees~~
- **Conclusion**

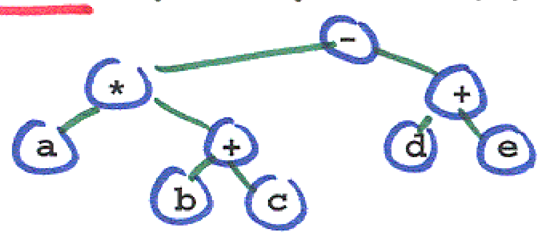
Other types of trees

- Need not have precisely two children
- Order might not matter

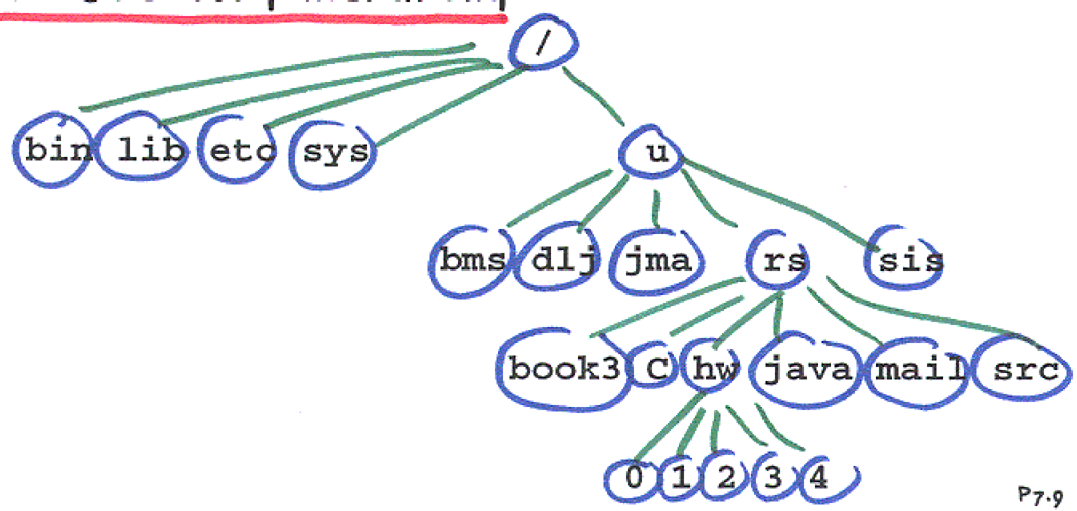
- "Family" tree



- Parse tree (a * (b + c)) - (d + e)



- UNIX directory hierarchy



What We Have Learned

- How to search and insert into:
  - sorted arrays
  - linked lists
  - binary search trees
- How long these operations take for the different data structures
- The meaning of different traversal orders and how the code for them works